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Effect of alternating electric field on Čerenkov radiation

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Abstract. The effects of an external alternating electric field on radiation by a charged particle traversing an infinite isotropic dielectric with a uniform velocity v_0 exceeding the phase velocity of light c/n, where n is the refractive index of the dielectric, are considered. In the first case the field $\boldsymbol{E} = E_0 \sin \omega_0 t$ is applied parallel to the direction of motion of the particle and in the second case it is applied perpendicular to it. Exact formulae for the energy loss due to Čerenkov and Doppler radiations are obtained and the loss is evaluated for different applied alternating fields. In both cases it is found that the Čerenkov radiation is reduced considerably. For the same field frequency and field strength, the parallel field reduces Čerenkov radiation more effectively than the perpendicular one. With the application of an external alternating electric field, Doppler radiation starts appearing, reaches a maximum and decreases as the field intensity increases.

1. Introduction

The phenomenon of Čerenkov radiation has been studied extensively, both theoretically and experimentally, since its discovery. Theoretical treatment is classical as well as quantum mechanical and the main interest lies in finding the energy loss by the charged particle. Ginzburg (1947) has indicated the possibility of obtaining microwave radiation from the passage of fast electrons moving very close to a dielectric of large refractive index when an alternating electric field is applied perpendicular to the direction of motion of the electrons. Since then many workers have tried to find out the effects of alternating fields on Čerenkov radiation. By applying the method of images Diasamidze and Tavdgiridze (1972) have shown that the intensity of Čerenkov radiation is reduced when an alternating electric field is applied parallel to the direction of motion of the charge moving close to the boundary separating two dielectrics. The effect of an intense monochromatic circularly polarised wave on the system of a fast electron moving through a dielectric is considered by Dementev *et al* (1972) and its effect on the intensity of Čerenkov radiation is studied by applying the classical theory of fields. The effects of external oscillating fields on Čerenkov radiation are discussed qualitatively and the case of an oscillating magnetic field is studied in detail by Mysakhanyan and Nikishov (1974). In this paper we have investigated the effects of an external alternating electric field on Čerenkov radiation in an infinite isotropic dielectric medium. We have, in particular, considered the interesting situation suggested by Ginzburg. In § 2 the field is applied parallel to the direction of motion of electrons and in § 3 it is applied perpendicular to it. In both the cases the intensities of Čerenkov and Doppler radiation are obtained. It is found that very weak alternating fields have practically no effect on Čerenkov radiation while moderately strong fields suppress it considerably. For the same field frequency and field strength, the parallel field reduces Čerenkov radiation more effectively than the perpendicular one. Doppler radiation starts appearing with the application of alternating electric field, attains a maximum value and then decreases.

2. Parallel field

Consider an electron moving with a uniform velocity $\boldsymbol{v}_0$ along the z axis through a medium characterised by dielectric constant ϵ and magnetic permeability μ. An alternating electric field $\boldsymbol{E}=\boldsymbol{E}_0 \sin \omega_0 t$ is applied parallel to $\boldsymbol{v}_0$. Maxwell's equations for the electromagnetic field inside the medium for this situation are

$$\nabla \cdot \boldsymbol{D}=4\pi\rho \tag{1a}$$

$$\nabla \times \boldsymbol{E}=\frac{-1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{1b}$$

$$\nabla \cdot \boldsymbol{B}=0 \tag{1c}$$

$$\nabla \times \boldsymbol{B}=\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}-\frac{4\pi}{c}\boldsymbol{j} \tag{1d}$$

where the electric induction $\boldsymbol{D}=\epsilon\boldsymbol{E}$ and the magnetic induction $\boldsymbol{B}=\mu\boldsymbol{H}$. Here

$$\rho=e\delta(x)\,\delta(y)\,\delta(z-z(t)) \tag{2}$$

$$\boldsymbol{j}=\rho\boldsymbol{v} \tag{3}$$

are the charge and current densities respectively. Further

$$\boldsymbol{v}=\boldsymbol{v}_0-\boldsymbol{v}_{\mathrm{u}}\cos\omega_0 t=v_z \tag{4}$$

$$z(t)=v_0 t-(v_{\mathrm{u}}/\omega_0)\sin\omega_0 t \tag{5}$$

where $v_{\mathrm{u}}=e\boldsymbol{E}_0/m\omega_0$. We write Maxwell's equations (1) as functions of vector and scalar potentials in the form

$$\nabla^2\phi-\frac{n^2}{c^2}\frac{\partial^2\phi}{\partial t^2}=\frac{-4\pi\rho}{\epsilon} \tag{6a}$$

$$\boldsymbol{E}=-\nabla\phi-\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \tag{6b}$$

$$\boldsymbol{B}=\nabla\times\boldsymbol{A} \tag{6c}$$

$$\nabla^2\boldsymbol{A}-\frac{n^2}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2}=\frac{-4\pi\mu}{c}\boldsymbol{j}. \tag{6d}$$

Here we have used the Lorentz gauge

$$\nabla\cdot\boldsymbol{A}+\frac{n^2}{c}\frac{\partial\phi}{\partial t}=0$$

and the refractive index $n=(\epsilon\mu)^{1/2}$. Comparison of equations for $\boldsymbol{A}$ and ϕ in equation (6) gives $\boldsymbol{A} = \boldsymbol{\beta} n^2 \phi$, where $\boldsymbol{\beta} = \boldsymbol{v}/c$. We find the solution of equation (6) in the form

$$\boldsymbol{A}(x, y, z, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \boldsymbol{A}(x, y, k_z, \omega)\, \mathrm{e}^{\mathrm{i}(k_z z - \omega t)}\, \mathrm{d}k_z\, \mathrm{d}\omega. \tag{7}$$

Using expansions of δ functions and the relation

$$\mathrm{e}^{\mathrm{i}a\sin\theta} = \sum_{l=-\infty}^{+\infty} J_l(a)\, \mathrm{e}^{\mathrm{i}l\theta}$$

where $J_l(a)$ is the Bessel function of the first kind, we obtain from equation (3):

$$j_z(x, y, z, t) = \frac{ev}{2\pi}\delta(x)\delta(y) \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}k_z\, \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}(k_z z - \omega t)}\, \delta(\omega + l\omega_0 - k_z v_0) J_l\left(\frac{k_z v_u}{\omega_0}\right). \tag{8}$$

Using equations (7) and (8) and transforming from the Cartesian coordinates (x, y, z) to cylindrical ones (ρ, ϕ, z), equation (6d) for $\boldsymbol{A}(\rho, k_z, \omega)$ takes the following form:

$$\frac{\partial^2 \boldsymbol{A}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \boldsymbol{A}}{\partial \rho} + s^2 \boldsymbol{A} = \frac{-\mu e \boldsymbol{v}}{\pi c \rho}\delta(\rho) \sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_z v_u}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0) \tag{9}$$

where $s^2 = (\omega^2 n^2/c^2) - k_z^2$. Here $\boldsymbol{A}(\rho, k_z, \omega)$ is a cylindrical function satisfying the Bessel equation everywhere except at the pole where $\rho = 0$. For velocities of the electron exceeding the phase velocity of light inside the medium, i.e. $v > c/n$ or $\beta n > 1$, equation (9) gives the following solution for the radiation field for $\omega > 0$:

$$\boldsymbol{A}(\rho, k_z, \omega) = \frac{-e\mu \boldsymbol{v}}{2c} N_0(s\rho) \sum_{l=-\infty}^{+\infty} J_l\left(\frac{k_z v_u}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0) \tag{10}$$

where $N_0(s\rho)$ is a Bessel function of the second kind, and a complex conjugate expression for $\omega < 0$. Using the asymptotic form of $N_0(s\rho)$ and keeping terms corresponding to outgoing waves only, equations (9) and (10) give:

$$\boldsymbol{A}(\rho, z, t) = \frac{\mathrm{i}e\mu \boldsymbol{v}}{c} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega\, \mathrm{d}k_z}{\sqrt{(2\pi s\rho)}}\, \mathrm{e}^{\mathrm{i}\chi} J_l\left(\frac{k_z v_u}{\omega_0}\right)\delta(\omega + l\omega_0 - k_z v_0) \tag{11}$$

where $\chi = k_z z - \omega t + s\rho - \frac{1}{4}\pi$. Integration with respect to k_z can be performed by using the δ function. Substituting equation (11) into equations (6a) and (6b) we get the following expressions for the electromagnetic field components:

$$E_z = \frac{-e\mu v^2}{v_0^2 c^2} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\omega + l\omega_0}{\sqrt{(2\pi s\rho)}}\, \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\chi}\left(\frac{\omega v_0}{(\omega + l\omega_0)v} - \frac{1}{\beta^2 n^2}\right) J_l\left(\frac{\omega + l\omega_0}{\omega_0 v_0} v_u\right) \tag{12}$$

$$E_\rho = \frac{e\mu}{v_0 v} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\omega\, \mathrm{d}\omega}{\sqrt{(2\pi s\rho)}}\, \mathrm{e}^{\mathrm{i}\chi}\, n^{-2} \sqrt{\left(\beta^2 n^2 - \frac{(\omega + l\omega_0)^2 v^2}{\omega^2 v_0^2}\right)} J_l\left(\frac{k_z v_u}{\omega_0}\right) \tag{13}$$

and

$$H_\phi = \frac{e}{c}\frac{v}{v_0} \int_{-\infty}^{+\infty} \frac{\sqrt{s}}{\sqrt{(2\pi\rho)}}\, \mathrm{e}^{\mathrm{i}\chi} J_l\left(\frac{k_z v_u}{\omega_0}\right) \mathrm{d}\omega \tag{14}$$

$$E_\phi = H_\rho = H_z = 0. \tag{15}$$

The total energy radiated by the electron through the surface of a cylinder of length l

whose axis coincides with the line of motion of the electron is equal to

$$W = 2\pi\rho l \int_{-\infty}^{+\infty} \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{H})_\rho \,\mathrm{d}t. \tag{16}$$

Using

$$\int_{-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}(\omega+\omega')t}\,\mathrm{d}t = 2\pi\delta(\omega+\omega')$$

and equations (12), (13), (14) and (15) we get

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \frac{e^2\mu}{c^2}\frac{v^3}{v_0^3}\sum_{l=-\infty}^{+\infty}\int_0^\infty \mathrm{d}\omega\,(\omega+l\omega_0)\left(\frac{\omega v_0}{(\omega+l\omega_0)v} - \frac{1}{\beta^2 n^2}\right)J_l^2\left(\frac{\omega+l\omega_0}{\omega_0 v_0}v_\mathrm{u}\right). \tag{17}$$

The expression for energy loss given by equation (17) is similar to that given in equation (24) of Diasamidze and Tavdgiridze (1972) apart from the corrections that the first term in the first large parentheses is not $[\omega/(\omega+l\omega_0)]^2$ but $[\omega/(\omega+l\omega_0)\,.\,v_0/v]$ and there is a multiplying factor of $(v/v_0)^3$ in equation (17) instead of $(v/v_0)^2$. Now (v/v_0) is a factor which depends on the intensity and frequency of the applied field. Here, since $v = v_0 \pm v_\mathrm{u}$ and $v_\mathrm{u} = eE_0/m\omega_0$, there is, in principle, no restriction to the field-dependent value of v_u. For the weak field $v_\mathrm{u} \ll v_0$ and $v/v_0 \sim 1$ so the case is rather uninteresting since the energy loss is practically unchanged. Experimentally it is possible to apply intense fields to the dielectric medium and we can get $v_\mathrm{u} \leqslant v_0$ and also $v_\mathrm{u} > v_0$. Now to analyse the results one should consider integrated values and not the integrands as is done in Diasamidze and Tavdgiridze (1972). Putting $\omega' = \omega + l\omega_0$ equation (17) becomes

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \frac{\mu e^2}{c^2}\frac{v^3}{v_0^3}\sum_{l=-\infty}^{+\infty}\left[\int_0^\infty\left(\frac{v_0}{v} - \frac{1}{\beta^2 n^2}\right)J_l^2(\lambda\omega')\,\mathrm{d}\omega' - l\frac{\omega_0 v_0}{v}\int_0^\infty J_l^2(\lambda\omega')\,\mathrm{d}\omega'\right] \tag{18}$$

where $\lambda = v_\mathrm{u}/\omega_0 v_0$. Neglecting dispersion and using the following integrals of Bessel functions

$$\int_0^\infty J_l^2(\lambda\omega)\,\mathrm{d}\omega = \frac{1}{\pi\lambda}$$

and

$$\int_0^\omega J_l^2(\lambda\omega)\,\mathrm{d}\omega\,\omega = \tfrac{1}{2}\omega^2(J_l^2(\lambda\omega) - J_{l-1}(\lambda\omega)J_{l+1}(\lambda\omega))$$

equation (18) becomes

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \frac{\mu e^2}{c^2}\frac{v^3}{v_0^3}\sum_{l=-\infty}^{+\infty}\left[\left(\frac{v_0}{v} - \frac{1}{\beta^2 n^2}\right)\frac{\omega_\mathrm{m}^2}{2}(J_l^2(\lambda\omega_\mathrm{m}) - J_{l-1}(\lambda\omega_\mathrm{m})J_{l+1}(\lambda\omega_\mathrm{m})) - \frac{l\omega_0 v_0}{\pi v\lambda}\right] \tag{19}$$

where ω_m denotes the upper limit for the frequency up to which the Čerenkov radiation condition, namely $\beta n(\omega) > 1$, can be satisfied. Equation (19) represents the radiation in the presence of an alternating electric field $\boldsymbol{E}$ applied parallel to the electron velocity v_0 satisfying $\beta n(\omega) > 1$. The terms with index $l = 0$ give Čerenkov radiation and those with $l \neq 0$ correspond to Doppler radiation. Positive integral values of index l give anomalous Doppler radiation which is emitted inside the Čerenkov cone while negative integral values of l contribute to normal Doppler radiation that appears outside the cone (Diasamidze and Tavdgiridze 1972, Frank

1943). Putting index $l=0$ in equation (19) we obtain the following expression for Čerenkov radiation:

$$\frac{\mathrm{d}W}{\mathrm{d}l}=\frac{\mu e^2}{c^2}\frac{v^3}{v_0^3}\left(\frac{v_0}{v}-\frac{1}{\beta^2 n^2}\right)\frac{\omega_{\mathrm{m}}^2}{2}(J_0^2(\lambda\omega_{\mathrm{m}})+J_1^2(\lambda\omega_{\mathrm{m}}))$$
$$=I_{\mathrm{C}}(J_0^2(\lambda\omega_{\mathrm{m}})+J_1^2(\lambda\omega_{\mathrm{m}})). \tag{20}$$

Equation (20) shows that the Čerenkov radiation in an alternating field has an oscillatory behaviour. In the absence of the field, equation (20) reduces to the familiar result given by Tamm and Frank (1937, 1967). The intensity of Čerenkov radiation can be found very easily from equation (20) for any values of the field parameters, namely E_0 and ω_0, for any region of interest wherever the Čerenkov radiation condition $\beta n(\omega)>1$ is satisfied.

3. Perpendicular field

Let us consider now the situation when $\boldsymbol{E}$ is perpendicular to $\boldsymbol{v}_0$. Let $\boldsymbol{E}$ be parallel to $\boldsymbol{x}$ and $\boldsymbol{v}_0$ be parallel to $\boldsymbol{z}$. In this case the current density

$$\boldsymbol{j}(x,y,z,t)=e\boldsymbol{v}(\delta(x-x(t))\delta(y)\delta(z-z(t)))$$

where

$$v_x=-v_{\mathrm{u}}\cos\omega_0 t, \qquad v_{\mathrm{u}}=eE_0/m\omega_0$$

$$v_z=v_0, \qquad v_y=0, \qquad x(t)=\frac{-v_{\mathrm{u}}}{\omega_0}\sin\omega_0 t$$

takes the following form:

$$\boldsymbol{j}(x,y,z,t)=\frac{e\boldsymbol{v}\delta(y)}{4\pi^2}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}k_x\,\mathrm{d}k_z\,\mathrm{d}\omega\,\mathrm{e}^{\mathrm{i}(k_x x+k_z z-\omega t)}$$
$$\times\delta(\omega+l\omega_0-k_z v_0)J_l\left(\frac{k_x v_{\mathrm{u}}}{\omega_0}\right). \tag{21}$$

Let the vector potential $\boldsymbol{A}$ be expanded as

$$\boldsymbol{A}(x,y,z,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\boldsymbol{A}(k_x,y,k_z,\omega)\,\mathrm{e}^{\mathrm{i}(k_x x+k_z z-\omega t)}\,\mathrm{d}k_x\,\mathrm{d}k_z\,\mathrm{d}\omega. \tag{22}$$

Substituting equations (21) and (22) in equation (6d) we get the following equation for the vector potential $\boldsymbol{A}(k_x,y,k_z,\omega)$:

$$\frac{\partial^2\boldsymbol{A}}{\partial y^2}+s'^2\boldsymbol{A}=\frac{-\mu e}{\pi c}\boldsymbol{v}\delta(y)\sum_{l=-\infty}^{+\infty}J_l\left(\frac{k_x v_{\mathrm{u}}}{\omega_0}\right)\delta(\omega+l\omega_0-k_z v_0) \tag{23}$$

where $s'^2=(\omega^2 n^2/c^2)-k_x^2-k_z^2$. Now the solution of equation (23) is given by the Green function. Considering only outgoing waves we get the solution for $\boldsymbol{A}$ as

$$\boldsymbol{A}(k_x,y,k_z,\omega)=\frac{-\mathrm{i}e\mu}{2\pi c}\boldsymbol{v}\sum_{l=-\infty}^{+\infty}J_l\left(\frac{k_x v_{\mathrm{u}}}{\omega_0}\right)\delta(\omega+l\omega_0-k_z v_0)\frac{\mathrm{e}^{\mathrm{i}s'y}}{s'} \tag{24}$$

for $\omega>0$, and a complex conjugate expression for $\omega<0$. Substituting $\boldsymbol{A}(k_x,y,k_z,\omega)$

from equation (24) into equation (22), we get

$$\boldsymbol{A}(x, y, z, t)=\frac{-\mathrm{i}e\mu}{2\pi c}\boldsymbol{v}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}k_x\,\mathrm{d}k_z\,\mathrm{d}\omega\frac{\mathrm{e}^{\mathrm{i}\chi'}}{s'}J_l\left(\frac{k_x v_\mathrm{u}}{\omega_0}\right)\delta(\omega+l\omega_0-k_z v_0), \tag{25}$$

where $\chi'=k_x x+k_z z-\omega t+s'y$. Using equations (25) and (6b) and expressing scalar potential ϕ in terms of the magnitude of vector potential $|\boldsymbol{A}|$ we get

$$E_z=\frac{e\mu}{2\pi c^2}\frac{v^2}{v_0^2}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{(\omega+l\omega_0)\,\mathrm{d}\omega\,\mathrm{d}k_z}{s'}\mathrm{e}^{\mathrm{i}\chi'}J_l\left(\frac{k_x v_\mathrm{u}}{\omega_0}\right)\left(\frac{\omega v_0^2}{v^2(\omega+l\omega_0)}-\frac{1}{\beta^2 n^2}\right). \tag{26}$$

Now the energy loss by the charge per unit path length is the same as the energy given out in the radiation. Hence

$$\begin{aligned}-\frac{\mathrm{d}W}{\mathrm{d}z}&=-eE_z|_{x=x(t),y=0,z=v_0t}\\&=\frac{-e^2\mu}{2\pi c^2}\frac{v^2}{v_0^2}\sum_{l=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\omega+l\omega_0)\,\mathrm{d}\omega\\&\quad\times\left(\frac{\omega v_0^2}{(\omega+l\omega_0)v^2}-\frac{1}{\beta^2n^2}\right)\frac{\mathrm{d}k_x}{s'(k_x,\omega)}J_l^2\left(\frac{k_x v_\mathrm{u}}{\omega_0}\right).\end{aligned} \tag{27}$$

Consider now the integral with respect to k_x in equation (27):

$$\begin{aligned}I&=\int_{-\infty}^{+\infty}\frac{\mathrm{d}k_x}{\sqrt{(b^2-k_x^2)}}J_l^2(\lambda' k_x)\\&=\frac{(\lambda' b/2)^{2l}\Gamma(1/2)\Gamma[(2l+1)/2]}{\Gamma^3(l+1)}{}_2F_3\left(\begin{matrix}\frac{1}{2}(2l+1),\frac{1}{2}(2l+1)\\2l+1,l+1,l+1\end{matrix}\middle|-\lambda'^2b^2\right)\end{aligned} \tag{28}$$

where

$$b^2=\frac{\omega^2n^2}{c^2}-k_z^2,\qquad \lambda'=\frac{v_\mathrm{u}}{\omega_0}\qquad\text{and}\qquad \mathrm{Re}\,(2l+1)>0.$$

Here ${}_2F_3$ denotes the hypergeometric function (Luke 1962).

Using equation (28) in equation (27) we get

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}z}&=\frac{e^2\mu}{\pi c^2}\frac{v^2}{v_0^2}\sum_{l=-\infty}^{+\infty}\frac{(\lambda'/2)^{2l}\Gamma(1/2)\Gamma[(2l+1)/2]}{\pi^3(l+1)}\int_0^\infty(\omega+l\omega_0)\,\mathrm{d}\omega\\&\quad\times\left(\frac{\omega}{\omega+l\omega_0}\frac{v_0^2}{v^2}-\frac{1}{\beta^2n^2}\right)b^{2l}(\omega){}_2F_3\left(\begin{matrix}\frac{1}{2}(2l+1),\frac{1}{2}(2l+1)^2\\{}'2l+1,l+1,l+1\end{matrix}\middle|-\lambda'^2b^2(\omega)\right).\end{aligned} \tag{29}$$

This is a general result giving radiation loss in the presence of a perpendicular alternating electric field. The loss due to Čerenkov radiation is obtained by putting $l=0$ in equation (29) while $l\neq 0$ corresponds to loss due to Doppler radiation. From equation (29) we get the following expression for Čerenkov radiation:

$$\frac{\mathrm{d}W}{\mathrm{d}z}=\frac{e^2\mu}{c^2}\frac{v^2}{v_0^2}\int_0^\infty\omega\,\mathrm{d}\omega\left(\frac{v_0^2}{v^2}-\frac{1}{\beta^2n^2}\right){}_2F_3\left(\begin{matrix}\frac{1}{2},\frac{1}{2}\\1,1,1\end{matrix}\middle|-\lambda'^2b^2(\omega)\right). \tag{30}$$

The upper limit of the above integral is replaced by ω_m, the maximum frequency up to which the condition for Čerenkov radiation, $\beta^2 n^2(\omega) > 1$, can be satisfied. Using expansion of hypergeometric function ${}_2F_3$, and neglecting dispersion (i.e. taking n = constant) and integrating term by term we get from equation (30):

$$I_{\mathrm{Cer}} = \left.\frac{\mathrm{d}W}{\mathrm{d}z}\right|_{l=0} = I'_{\mathrm{C}}\left(1 - \frac{\beta_0^2 n^2 - 1}{8}(\lambda\omega_m)^2 + \frac{3(\beta_0^2 n^2 - 1)^2}{256}(\lambda\omega_m)^4 - \frac{25}{36864}(\beta_0^2 n^2 - 1)^3(\lambda\omega_m)^6 + \ldots\right) \tag{31}$$

where

$$I'_{\mathrm{C}} = \frac{e^2\mu}{c^2}\frac{v^2}{v_0^2}\left(\frac{v_0^2}{v^2} - \frac{1}{\beta^2 n^2}\right)\frac{\omega_m^2}{2}, \qquad \lambda' = \lambda v_0.$$

Expression (31) gives the intensity of Čerenkov radiation in the presence of a perpendicular alternating electric field. When $E_0 = 0$ equation (31) reduces to the familiar result of Tamm and Frank (1937, 1967). Here l takes only positive integral values, $l = +1, +2, \ldots$, and corresponding terms in equation (29) give anomalous Doppler radiation, while normal Doppler radiation is forbidden in this case. Putting $l = +1$ in (29) we get the following expression for the first mode of Doppler radiation:

$$I_{\mathrm{Dop}} = \left.\frac{\mathrm{d}W}{\mathrm{d}z}\right|_{l=+1} = \frac{e^2\mu}{c^2}\frac{v^2}{v_0^2}\frac{\lambda'^2}{8}\int_0^\infty (\omega + \omega_0)\,\mathrm{d}\omega\left(\frac{\omega}{\omega + \omega_0}\frac{v_0^2}{v^2} - \frac{1}{\beta^2 n^2}\right) \times b^2(\omega)\,{}_2F_3\left(\begin{matrix}\frac{3}{2}, \frac{3}{2}\\ 3, 2, 2\end{matrix}\,\middle|\, -\lambda'^2 b^2(\omega)\right). \tag{32}$$

Expanding the hypergeometric function ${}_2F_3$ in equation (32), integrating term by term and using the fact that $\omega_m \gg \omega_0$, we get

$$I_{\mathrm{Dop}} = I'_{\mathrm{C}}\tfrac{1}{16}(\lambda\omega_m)^2(\beta_0^2 n^2 - 1)[1 - \tfrac{1}{8}(\beta_0^2 n^2 - 1)(\lambda\omega_m)^2 + \tfrac{25}{3072}(\beta_0^2 n^2 - 1)^2(\lambda\omega_m)^4 - \tfrac{49}{147456}(\beta_0^2 n^2 - 1)^3(\lambda\omega_m)^6 + \ldots] \tag{33}$$

where I'_{C} is the same as given above.

By using equations (19) with $l = +1$, (20), (31) and (33) we have calculated intensities of Čerenkov radiation and Doppler radiation for a particular case of $\beta = 0{\cdot}998$, $n = 1{\cdot}221$ for various values of $\lambda\omega_m$ which depend on field parameters E_0 and ω_0. The results of the calculations are shown graphically in figure 1.

The effect of an alternating electric field is to suppress Čerenkov radiation. Reduction is slight when the field is weak while for moderately strong fields the reduction is considerable. For the same value of field parameters, the parallel field reduces Čerenkov radiation more effectively than the perpendicular one. With the application of an alternating field, Doppler radiation starts appearing and varies with parallel and perpendicular external fields as is shown in figure 1.

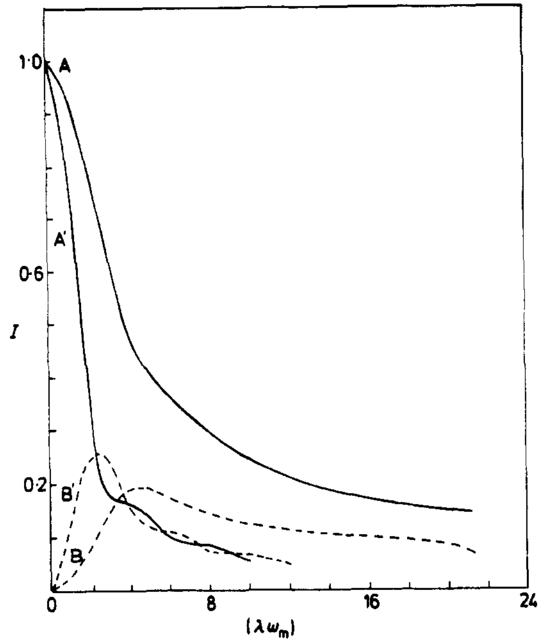

Figure 1. Variation of intensity of Čerenkov radiation (shown by A and A′ curves) and Doppler radiation (shown by B and B′ curves) with $\lambda\omega_m = eE_0\omega_m/m\omega_0^2 v_0$. For curves A and B, $\boldsymbol{E}$ is perpendicular to $\boldsymbol{v}_0$ and I is in units of I_C' while for A′ and B′, $\boldsymbol{E}$ is parallel to $\boldsymbol{v}_0$ and I is in units of I_C.

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